

Summations - Closed Forms

$$(1) \sum_{i=1}^n c = (n-1)c + c$$

$$(2) \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$(3) \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(4) \sum_{i=0}^n a^i = \frac{a^{n+1}-1}{a-1} \quad (\text{where } a \neq 1)$$

$$(5) \sum_{i=1}^n ka^i = \frac{a-(n+1)a^{n+1}+na^{n+2}}{(a-1)^2} \quad (\text{where } a \neq 1)$$

Summations - Rules

$$(1) \sum_{i=1}^n c a_i = c \sum_{i=1}^n a_i \quad (2) \sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$$

$$(3) \sum_{i=1}^n c a_i d^i = c \sum_{i=1}^n a_i d^i \quad (4) \sum_{i=1}^n a_i b_i = \sum_{i=1}^n a_i b_i$$

$$(5) \text{Collapsing Summation} \quad \sum_{i=1}^n (a_i - a_{i-1}) = a_n - a_0 \quad \text{and} \quad \sum_{i=1}^n (a_{i-1} - a_i) = a_0 - a_n$$

Logarithm Rules

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b(x^a) = a \cdot \log_b(x)$$

$$\log_k(x) = \frac{\ln(x)}{\ln(k)} = \frac{\log_{10}(x)}{\log_{10}(k)}$$

$$a^{\log_b k} = k^{\log_b a}$$

Note: for AoA, $\lg = \log_2$

$$e.g. \lg 3 = \frac{\log_{10}(3)}{\log_{10}(2)}$$

Asymptotic Analysis - Common Orders of Growth

Slowest Growth - Fastest Growth

$\Theta(1)$: constant
 $\Theta(\log n)$: logarithmic
 $\Theta(n)$: linear
 $\Theta(n \log n)$:
 $\Theta(n^2)$: quadratic
 $\Theta(n^k)$ (for constant k) : polynomial
 $\Theta(k^n)$ (for constant k) : exponential

Master Theorem Shortcut

Case	Condition	Result
1	$k < E$	n^E
2	$k = E$	$n^E \lg(n)$
3	$k > E$	n^k

Fermat's Little Theorem

For any prime p , for any x :

$$x^p \equiv x \pmod{p}$$

Alternatively, for any $x \neq 0$:

$$x^{p-1} \equiv 1 \pmod{p}$$

Mod Operations

$$(x + y) \bmod n = ((x \bmod n) + (y \bmod n)) \bmod n$$

$$(xy) \bmod n = ((x \bmod n) \times (y \bmod n)) \bmod n$$

$$(x - y) \bmod n = ((x \bmod n) - (y \bmod n)) \bmod n$$