# Integral Trigonometry Cheat Sheet

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## Basic Trigonometric Integrals

| | |
|---|---|
| **$\int \sin(x)dx$** | **$-\cos(x)+C$** |
| **$\int \cos(x)dx$** | **$\sin(x)+C$** |
| **$\int \sec^2(x)dx$** | **$\tan(x)+C$** |
| **$\int \sec(x)\tan(x)dx$** | **$\sec(x)+C$** |
| $\int \csc^2(x)dx$ | $-\cot(x)+C$ |
| $\int \csc(x)\cot(x)dx$ | $-\csc(x)+C$ |

## Common Trigonometric Integrals

| | |
|---|---|
| **$\int \sin(2x)dx$** | **$-½\cos(2x)+C$** |
| **$\int \cos(2x)dx$** | **$½\sin(2x)+C$** $= \sin(x)\cos(x)+C$ |
| $\int \tan(x)dx$ | $\ln\lvert\sec(x)\rvert+C$ |
| $\int \sec(x)dx$ | $\ln\lvert\sec(x)+\tan(x)\rvert+C$ |
| $\int \sec^3(x)dx$ | $½(\sec(x)\tan(x)+\ln\lvert\sec(x)+\tan(x)\rvert)+C$ |
| $\int \csc(x)dx$ | $-\ln\lvert\csc(x)+\cot(x)\rvert+C$ |
| $\int \csc^3(x)dx$ | $-½(\csc(x)\cot(x)+\ln\lvert\csc(x)+\cot(x)\rvert)+C$ |
| **$\int 1/(1+x^2)dx$** | **$\arctan(x)+C$** |
| $\int 1/(a^2+x^2)dx$ | $(1/a)\arctan(x/a)+C$ |

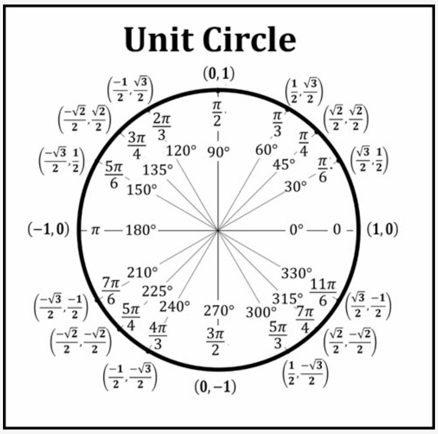
## Secant and Cosecant Integral Forms

| | |
|---|---|
| Secant Integral 1 | $\int \sec(x)dx = \ln\lvert\sec(x)+\tan(x)\rvert+C$ |
| Secant Integral 2 | $\int \sec(x)dx = -\ln\lvert\sec(x)-\tan(x)\rvert+C$ |
| Secant Integral 3 | $\int \sec(x)dx = ½\ln\lvert(\sin(x)+1)/(\sin(x)-1)\rvert+C$ |
| Secant Integral 4 | $\int \sec(x)dx = \ln\lvert\tan(x/2+\pi/4)\rvert+C$ |

## Secant and Cosecant Integral Forms (cont)

| | |
|---|---|
| Cosecant Integral 1 | $\int \csc(x)dx = -\ln\lvert\csc(x)+\cot(x)\rvert+C$ |
| Cosecant Integral 2 | $\int \csc(x)dx = \ln\lvert\csc(x)-\cot(x)\rvert+C$ |
| Cosecant Integral 3 | $\int \csc(x)dx = ½\ln\lvert(\cos(x)-1)/(\cos(x)+1)\rvert+C$ |
| Cosecant Integral 4 | $\int \csc(x)dx = \ln\lvert\tan(x/2)\rvert+C$ |

## Sine and Cosine Unit Circle




## Powers of Trigonometric Functions

https://cheatography.com/crossant/cheat-sheets/integral-cases-for-trigonometric-powers/

## Quotient and Reciprocal Identities

| | |
|---|---|
| **Tangent Quotient** | **$\tan(x)=\sin(x)/\cos(x)$** |
| **Cotangent Quotient** | **$\cot(x)=\cos(x)/\sin(x)$** |
| | |
| Sine Reciprocal | $\sin(x)=1/\csc(x)$ |
| Cosine Reciprocal | $\cos(x)=1/\sec(x)$ |
| Tangent Reciprocal | $\tan(x)=1/\cot(x)$ |
| **Cosecant Reciprocal** | **$\csc(x)=1/\sin(x)$** |
| **Secant Reciprocal** | **$\sec(x)=1/\cos(x)$** |
| Cotangent Reciprocal | $\cot(x)=1/\tan(x)$ |

## Pythagorean Identities

| | |
|---|---|
| Sine-Cosine Pythagorean | **$\sin^2(x)+\cos^2(x)=1$** |
| Sine Pythagorean | **$\sin^2(x)=1-\cos^2(x)$** |
| Cosine Pythagorean | **$\cos^2(x)=1-\sin^2(x)$** |
| Secant Pythagorean | **$\tan^2(x)+1=\sec^2(x)$** |
| Tangent Pythagorean | **$\tan^2(x)=\sec^2(x)-1$** |
| Secant-Tangent Pythagorean | $\sec^2(x)-\tan^2(x)=1$ |
| Cosecant Pythagorean | $1+\cot^2(x)=\csc^2(x)$ |
| Cotangent Pythagorean | $\cot^2(x)=\csc^2(x)-1$ |
| Cosecant-Cotangent Pythagorean | $\csc^2(x)-\cot^2(x)=1$ |

The last two triplets of Pythagorean identities are obtained by dividing all the terms of the original identity by $\sin^2(x)$ or $\cos^2(x)$

## Sum, Difference, and Product Identities

$\sin(x+y)=\sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x-y)=\sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y) - \sin(x)\sin(y)$

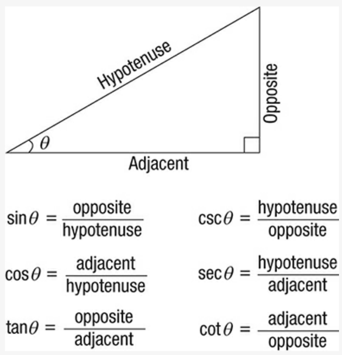
$\cos(x-y)=\cos(x)\cos(y) + \sin(x)\sin(y)$

$\sin(x)\sin(y)=½(\cos(x-y) - \cos(x+y))$

$\cos(x)\cos(y)=½(\cos(x+y) + \cos(x-y))$

sin(x)cosy)=½(sin(x+y) + sin(x-y))

## Right-Triangle Trigonometric Relations



## Trigonometric Substitutions

| | | |
|---|---|---|
| **$a^2$-$u^2$** | **Let u=asin(θ), du=acos(θ)dθ** | θ∈[-π/2, π/2] |
| **$u^2$-$a^2$** | **Let u=asec(θ), du=asec(θ)tan(θ)dθ** | θ∈[0, π/2) |
| **$u^2$+$a^2$** | **Let u=atan(θ), du=a$sec^2$(θ)dθ** | θ∈(-π/2, π/2) |
| $a^2$-$b^2u^2$ | Let u=(a/b)sin(θ), du=(a/b)cos(θ)dθ | θ∈[-π/2, π/2] |
| $b^2u^2$-$a^2$ | Let u=(a/b)sec(θ), du=(a/b)sec(θ)tan(θ)dθ | θ∈[0, π/2) |
| $b^2u^2$+$a^2$ | Let u=(a/b)tan(θ), du=(a/b)$sec^2$(θ)dθ | θ∈(-π/2, π/2) |

Trigonometric substitutions are typically used under radicals, however, they are not required to be

For definite integrals, you will need to set u equal to its respective bounds, and solve for θ in order to properly change the bounds of integration with respect to θ

## Half-Angle and Double-Angle Identities

| | |
|---|---|
| Sine Half-Angle | sin(x/2)=√(½(1-cos(x))) |
| Cosine Half-Angle | cos(x/2)=√(½(1+cos(x))) |
| **Sine Power-Reducing** | **$sin^2$(x)=½(1-cos(2x))** |
| **Cosine Power-Reducing** | **$cos^2$(x)=½(1+cos(2x))** |
| | |
| **Sine Double-Angle** | **sin(2x)=2sin(x)cos(x)** |
| **Cosine Double-Angle 1** | **cos(2x)=$cos^2$(x)-$sin^2$(x)** |
| Cosine Double-Angle 2 | cos(2x)=2$cos^2$(x)-1 |

## Half-Angle and Double-Angle Identities (cont)

| | |
|---|---|
| Cosine Double-Angle 3 | cos(2x)=1-2$sin^2$(x) |

Sine Power-Reducing and Cosine Power-Reducing identities are variations of the Half-Angle identities

## Tangent Unit Circle