

Basic Trigonometric Integrals

$\int \sin(x) dx$	$-\cos(x) + C$
$\int \cos(x) dx$	$\sin(x) + C$
$\int \sec^2(x) dx$	$\tan(x) + C$
$\int \sec(x) \tan(x) dx$	$\sec(x) + C$
$\int \csc^2(x) dx$	$-\cot(x) + C$
$\int \csc(x) \cot(x) dx$	$-\csc(x) + C$

Common Trigonometric Integrals

$\int \sin(2x) dx$	$-\frac{1}{2} \cos(2x) + C$
$\int \cos(2x) dx$	$\frac{1}{2} \sin(2x) + C = \sin(x) \cos(x) + C$
$\int \tan(x) dx$	$\ln \sec(x) + C$
$\int \sec(x) dx$	$\ln \sec(x) + \tan(x) + C$
$\int \sec^3(x) dx$	$\frac{1}{2}(\sec(x) \tan(x) + \ln \sec(x) + \tan(x)) + C$
$\int \csc(x) dx$	$-\ln \csc(x) + \cot(x) + C$
$\int \csc^3(x) dx$	$-\frac{1}{2}(\csc(x) \cot(x) + \ln \csc(x) - \cot(x)) + C$
$\int 1/(1+x^2) dx$	$\arctan(x) + C$
$\int 1/(a^2+x^2) dx$	$(1/a) \arctan(x/a) + C$

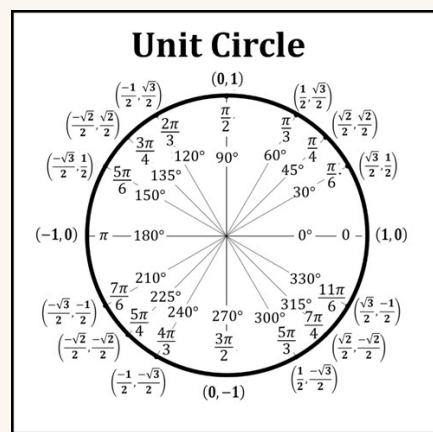
Secant and Cosecant Integral Forms

Secant Integral 1	$\int \sec(x) dx = \ln \sec(x) + \tan(x) + C$
Secant Integral 2	$\int \sec(x) dx = -\ln \sec(x) - \tan(x) + C$
Secant Integral 3	$\int \sec(x) dx = \frac{1}{2} \ln (\sin(x)+1)/(\sin(x)-1) + C$
Secant Integral 4	$\int \sec(x) dx = \ln \tan(x/2) + 2 + \pi/4 + C$

Secant and Cosecant Integral Forms (cont)

Cosecant Integral 1	$\int \csc(x) dx = -\ln \csc(x) + \cot(x) + C$
Cosecant Integral 2	$\int \csc(x) dx = \ln \csc(x) - \cot(x) + C$
Cosecant Integral 3	$\int \csc(x) dx = \frac{1}{2} \ln (\cos(x)-1)/(\cos(x)+1) + C$
Cosecant Integral 4	$\int \csc(x) dx = \ln \tan(x/2) + C$

Sine and Cosine Unit Circle



Powers of Trigonometric Functions

<https://cheatography.com/crossant/cheat-sheets/integral-cases-for-trigonometric-powers/>

Quotient and Reciprocal Identities

Tangent Quotient	$\tan(x) = \sin(x)/\cos(x)$
Cotangent Quotient	$\cot(x) = \cos(x)/\sin(x)$
Sine Reciprocal	$\sin(x) = 1/\csc(x)$
Cosine Reciprocal	$\cos(x) = 1/\sec(x)$
Tangent Reciprocal	$\tan(x) = 1/\cot(x)$
Cosecant Reciprocal	$\csc(x) = 1/\sin(x)$
Secant Reciprocal	$\sec(x) = 1/\cos(x)$
Cotangent Reciprocal	$\cot(x) = 1/\tan(x)$

Pythagorean Identities

Sine-Cosine Pythagorean	$\sin^2(x) + \cos^2(x) = 1$
Sine Pythagorean	$\sin^2(x) = 1 - \cos^2(x)$
Cosine Pythagorean	$\cos^2(x) = 1 - \sin^2(x)$
Secant Pythagorean	$\tan^2(x) + 1 = \sec^2(x)$
Tangent Pythagorean	$\tan^2(x) = \sec^2(x) - 1$
Secant-Tangent Pythagorean	$\sec^2(x) - \tan^2(x) = 1$
Cosecant Pythagorean	$1 + \cot^2(x) = \csc^2(x)$
Cotangent Pythagorean	$\cot^2(x) = \csc^2(x) - 1$
Cosecant-Cotangent Pythagorean	$\csc^2(x) - \cot^2(x) = 1$

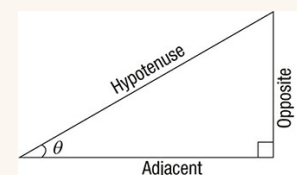
The last two triplets of Pythagorean identities are obtained by dividing all the terms of the original identity by $\sin^2(x)$ or $\cos^2(x)$

Sum, Difference, and Product Identities

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\sin(x)\sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y))$
$\cos(x)\cos(y) = \frac{1}{2}(\cos(x+y) + \cos(x-y))$
$\sin(x)\cos(y) = \frac{1}{2}(\sin(x+y) + \sin(x-y))$

Right-Triangle Trigonometric Relations



$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$
$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$	$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$

Trigonometric Substitutions

$a^2 - u^2$	Let $u = a \sin(\theta)$, $du = a \cos(\theta) d\theta$	$\theta \in [-\pi/2, \pi/2]$
$u^2 - a^2$	Let $u = a \sec(\theta)$, $du = a \sec(\theta) \tan(\theta) d\theta$	$\theta \in [0, \pi/2)$
$u^2 + a^2$	Let $u = a \tan(\theta)$, $du = a \sec^2(\theta) d\theta$	$\theta \in (-\pi/2, \pi/2)$
$a^2 - b^2 u^2$	Let $u = (a/b) \sin(\theta)$, $du = (a/b) \cos(\theta) d\theta$	$\theta \in [-\pi/2, \pi/2]$
$b^2 u^2 - a^2$	Let $u = (a/b) \sec(\theta)$, $du = (a/b) \sec(\theta) \tan(\theta) d\theta$	$\theta \in [0, \pi/2)$
$b^2 u^2 + a^2$	Let $u = (a/b) \tan(\theta)$, $du = (a/b) \sec^2(\theta) d\theta$	$\theta \in (-\pi/2, \pi/2)$

Trigonometric substitutions are typically used under radicals, however, they are not required to be

For definite integrals, you will need to set u equal to its respective bounds, and solve for θ in order to properly change the bounds of integration with respect to θ

Half-Angle and Double-Angle Identities

Sine Half-Angle	$\sin(x/2) = \sqrt{\frac{1}{2}(1 - \cos(x))}$
Cosine Half-Angle	$\cos(x/2) = \sqrt{\frac{1}{2}(1 + \cos(x))}$
Sine Power-Reducing	$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
Cosine Power-Reducing	$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
Sine Double-Angle	$\sin(2x) = 2 \sin(x) \cos(x)$
Cosine Double-Angle 1	$\cos(2x) = \cos^2(x) - \sin^2(x)$
Cosine Double-Angle 2	$\cos(2x) = 2 \cos^2(x) - 1$

Half-Angle and Double-Angle Identities (cont)

Cosine Double-Angle $\cos(2x) = 1 - 2 \sin^2(x)$

Sine Power-Reducing and Cosine Power-Reducing identities are variations of the Half-Angle identities

Tangent Unit Circle

