

### Geometric Mean

Best explained as an example, The geometric mean of  $a = 9$  and  $b = 4$  is 6, because  $6 = \sqrt{9 \cdot 4}$ .

### Geometric Mean Right Triangle Theorems

**Theorems Right Triangle Geometric Mean Theorems**

**8.2 Geometric Mean (Altitude) Theorem** The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

**Example** If  $\overline{CD}$  is the altitude to hypotenuse  $\overline{AB}$  of right  $\triangle ABC$ , then  $\frac{a}{h} = \frac{h}{b}$  or  $h = \sqrt{ab}$ .

**8.3 Geometric Mean (Leg) Theorem** The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

**Example** If  $\overline{CD}$  is the altitude to hypotenuse  $\overline{AB}$  of right  $\triangle ABC$ , then  $\frac{c}{a} = \frac{a}{x}$  or  $a = \sqrt{cx}$  and  $\frac{c}{b} = \frac{b}{y}$  or  $b = \sqrt{cy}$ .

### Pythagorean Theorem

a squared + b squared = c squared (Only for Right Triangles)

### 30-90-60 Triangles and 45-90-45 Triangles

**REVIEW: Special Right Triangles**

**45-45-90 isosceles right triangle**

Leg:  $x$ ,  $x$ ; Hypotenuse:  $x\sqrt{2}$

**30-60-90 special right triangle**

Short Leg:  $x$ ; Long Leg:  $x\sqrt{3}$ ; Hypotenuse:  $2x$

### Soh Cah Toa

|       |                       |
|-------|-----------------------|
| Sin() | Opposite / Hypotenuse |
| Cos() | Adjacent / Hypotenuse |
| Tan() | Opposite / Adjacent   |

### EXAMPLE I CANT EXPLAIN

**Example 3 Use Two Angles of Elevation or Depression**

**TREE REMOVAL** To estimate the height of a tree she wants removed, Mrs. Long sights the tree's top at a  $70^\circ$  angle of elevation. She then steps back 10 meters and sights the top at a  $26^\circ$  angle. If Mrs. Long's line of sight is 1.7 meters above the ground, how tall is the tree to the nearest meter?

**Understand**  $\triangle ABC$  and  $\triangle ABD$  are right triangles. The height of the tree is the sum of Mrs. Long's height and  $AB$ .

**Plan** Since her initial distance from the tree is not given, write and solve a system of equations using both triangles. Let  $AB = x$  and  $CB = y$ . So  $DB = y + 10$  and the height of the tree is  $x + 1.7$ .

### Continued

**Solve** Use  $\triangle ABC$ .

$$\tan 70^\circ = \frac{x}{y} \quad \tan = \frac{\text{opposite}}{\text{adjacent}} \quad m\angle ACB = 70^\circ$$

$$y \tan 70^\circ = x \quad \text{Multiply each side by } y.$$

Use  $\triangle ABD$ .

$$\tan 26^\circ = \frac{x}{y + 10} \quad \tan = \frac{\text{opposite}}{\text{adjacent}} \quad m\angle CDB = 26^\circ$$

$$(y + 10) \tan 26^\circ = x \quad \text{Multiply each side by } y + 10.$$

Substitute the value for  $x$  from  $\triangle ABD$  in the equation for  $\triangle ABC$  and solve for  $y$ .

$$y \tan 70^\circ = x$$

$$y \tan 70^\circ = (y + 10) \tan 26^\circ$$

$$y \tan 70^\circ = y \tan 26^\circ + 10 \tan 26^\circ$$

$$y \tan 70^\circ - y \tan 26^\circ = 10 \tan 26^\circ$$

$$y(\tan 70^\circ - \tan 26^\circ) = 10 \tan 26^\circ$$

$$y = \frac{10 \tan 26^\circ}{\tan 70^\circ - \tan 26^\circ}$$

Use a calculator to find that  $y \approx 2.16$ . Using the equation from  $\triangle ABC$ ,  $x = 2.16 \tan 70^\circ$  or about 5.9.

The height of the tree is  $5.9 + 1.7$  or 7.6, which is about 8 meters.

**Check** Substitute the value for  $y$  in the equation from  $\triangle ABD$ .

$$x = (2.16 + 10) \tan 26^\circ \text{ or about } 5.9. \text{ This is the same value found using the equation from } \triangle ABC. \checkmark$$

### Law of Sines

**Law of Sines**  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

### Law of Cosines

**Law of Cosines** Let  $\triangle ABC$  be any triangle with  $a$ ,  $b$ , and  $c$  representing the measures of the sides opposite the angles with measures  $A$ ,  $B$ , and  $C$ , respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A \quad b^2 = a^2 + c^2 - 2ac \cos B \quad c^2 = a^2 + b^2 - 2ab \cos C$$