

Parabola $f(x) = a(x-h)^2$

- $(-3-h)^2$ *h determines x axis*
- (2 negatives= positive) **If a is positive** parabola is **up or to the right**
- Vertex is at **(h,k)** **if a is negative** opens **left or down**
- $(x-3)^2 - 5 \rightarrow$ move to the right
- If parenthesis is addition, move to the left

Graph the functions. Plot at least 3..

Make a table for equation

pick points for x

Solve

graph answers

should be in the form of $f(x) = b^x$

Graph the functions. plot at least.. (Rules)

- if $b > 1$ its a exponential growth function
- if $0 < b < 1$ its a exponential decay function
- if $b > 1$ Domain = $(-\infty, \infty)$
- if $b > 1$ Range = $(0, \infty)$
- if $b > 1$ The line $y=0$ is horizontal asymptote
- if $b > 1$ Function passes through $(0, 1)$

Solve. $ax^2 + bx + c = 0$

OBJECTIVE

2 Solving by Completing the Square

Notice from Examples 3 and 4 that, if we write a quadratic equation so that one side is the square of a binomial, we can solve by using the square root property. To write the square of a binomial, we write perfect square trinomials. Recall that a perfect square trinomial is a trinomial that can be factored into two identical binomial factors.

Perfect Square Trinomials	Factored Form
$x^2 + 8x + 16$	$(x + 4)^2$
$x^2 - 6x + 9$	$(x - 3)^2$
$x^2 + 3x + \frac{9}{4}$	$(x + \frac{3}{2})^2$

Notice that for each perfect square trinomial in x , the constant term of the trinomial is the square of half the coefficient of the x -term. For example,

$$\begin{array}{ccc} x^2 + 8x + 16 & & x^2 - 6x + 9 \\ \swarrow \quad \searrow & & \swarrow \quad \searrow \\ \frac{1}{2}(8) = 4 \text{ and } 4^2 = 16 & & \frac{1}{2}(-6) = -3 \text{ and } (-3)^2 = 9 \end{array}$$

The process of writing a quadratic equation so that one side is a perfect square trinomial is called **completing the square**.

EXAMPLE 5 Solve $p^2 + 2p = 4$ by completing the square.

Solution First, add the square of half the coefficient of p to both sides so that the resulting trinomial will be a perfect square trinomial. The coefficient of p is 2.

$$\frac{1}{2}(2) = 1 \text{ and } 1^2 = 1$$

Add 1 to both sides of the original equation.

$$\begin{array}{l} p^2 + 2p = 4 \\ p^2 + 2p + 1 = 4 + 1 \quad \text{Add 1 to both sides.} \\ (p + 1)^2 = 5 \quad \text{Factor the trinomial; simplify the right side.} \end{array}$$

We may now use the square root property and solve for p .

$$\begin{array}{l} p + 1 = \pm \sqrt{5} \quad \text{Use the square root property.} \\ p = -1 \pm \sqrt{5} \quad \text{Subtract 1 from both sides.} \end{array}$$

Notice that there are two solutions: $-1 + \sqrt{5}$ and $-1 - \sqrt{5}$.

REMARK

more parabolas

OBJECTIVE

3 Graphing $f(x) = (x - h)^2 + k$

As we will see in graphing functions of the form $f(x) = (x - h)^2 + k$, it is possible to combine vertical and horizontal shifts.

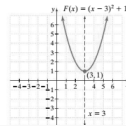
Graphing the Parabola Defined by $f(x) = (x - h)^2 + k$

The parabola has the same shape as $y = x^2$. The vertex is (h, k) , and the axis of symmetry is the vertical line $x = h$.

EXAMPLE 5 Graph: $F(x) = (x - 3)^2 + 1$

Solution The graph of $F(x) = (x - 3)^2 + 1$ is the graph of $y = x^2$ shifted 3 units to the right and 1 unit up. The vertex is then $(3, 1)$, and the axis of symmetry is $x = 3$. A few ordered pair solutions are plotted to aid in graphing.

x	F(x) = (x - 3) ² + 1
1	5
2	2
4	2
5	5



Quadratic functions of the form

Graphing the Parabola Defined by $f(x) = ax^2$

If a is positive, the parabola opens upward, and if a is negative, the parabola opens downward. If $|a| > 1$, the graph of the parabola is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph of the parabola is wider than the graph of $y = x^2$.

Shifting Parabolas

- $f(x) = ax^2 + bx + c$ is a *parabola*
- $a > 0$ opens up
- $a < 0$ opens down
- $f(x) = x^2 + k$ is a vertical shift of $f(x) = x^2$
- $h > 0$ shifted right vice versa
- $f(x) = (x - h)^2$

x (possible extra terms here) = square root of b

Square Root Property

If b is a real number and if $a^2 = b$, then $a = \pm \sqrt{b}$.

Helpful Hint

The notation ± 3 , for example, is read as "plus or minus 3." It is a shorthand notation for the pair of numbers $+3$ and -3 .

EXAMPLE 1 Use the square root property to solve $x^2 = 50$.

Solution

$$\begin{array}{l} x^2 = 50 \\ x = \pm \sqrt{50} \quad \text{Use the square root property.} \\ x = \pm 5\sqrt{2} \quad \text{Simplify the radical.} \end{array}$$

Check:

Let $x = 5\sqrt{2}$.	Let $x = -5\sqrt{2}$.
$x^2 = 50$	$x^2 = 50$
$(5\sqrt{2})^2 \stackrel{?}{=} 50$	$(-5\sqrt{2})^2 \stackrel{?}{=} 50$
$25 \cdot 2 \stackrel{?}{=} 50$	$25 \cdot 2 \stackrel{?}{=} 50$
$50 = 50$ True	$50 = 50$ True

The solutions are $5\sqrt{2}$ and $-5\sqrt{2}$, or the solution set is $\{-5\sqrt{2}, 5\sqrt{2}\}$.

Steps: get it in the form of the equation by adding and subtracting different sides
Apply square root and the plus or minus sign

Find the inverse

Change $f(x)$ to y

Switch x & y

Solve for y

Don't forget about cross multiplying

Logarithms

$\log_{1=1}$ Only solve multiple logs if they have the same base

$\log \log(xy) = \log(x) + \log(y)$

$b^x = x$

$b^{\log \text{ sub } bx} = x$

$\log 1 = 0$

$\log b = 1$

power is what it's equal to
base is the sub

Solve log equation.

Convert to exponential form based off of note

Simplify

$$2^3 = 8 \text{ which is } \log 8 = 3$$

Solve using Substitution.

Substitute the same terms with a letter

Solve for the letter ex: $(x=2)$

Replace the letter with what was in the equation

Solve

C

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Solve the inequality using the test point method

The following steps may be used to solve a rational inequality with variables in the denominator.

Solving a Rational Inequality

Step 1. Solve for values that make all denominators 0.

Step 2. Solve the related equation.

Step 3. Separate the number line into regions with the solutions from Steps 1 and 2.

Step 4. For each region, choose a test point and determine whether its value satisfies the original inequality.

Step 5. The solution set includes the regions whose test point value is a solution. Check whether to include values from Step 2. Be sure *not* to include values that make any denominator 0.

EXAMPLE 5 Solve: $\frac{5}{x+1} < -2$

Solution First we find values for x that make the denominator equal to 0.

$$x + 1 = 0$$

$$x = -1$$

Next we solve $\frac{5}{x+1} = -2$.

$$(x+1) \cdot \frac{5}{x+1} = (x+1) \cdot -2 \quad \text{Multiply both sides by the LCD, } x+1.$$

$$5 = -2x - 2 \quad \text{Simplify.}$$

$$7 = -2x$$

$$-\frac{7}{2} = x$$



We use these two solutions to divide a number line into three regions and choose test points. Only a test point value from region B satisfies the original inequality. The solution set is $(-\frac{7}{2}, -1)$, and its graph is shown.



Solving a Polynomial Inequality EX

EXAMPLE 1 Solve: $(x+3)(x-3) > 0$

Solution First we solve the related equation, $(x+3)(x-3) = 0$.

$$(x+3)(x-3) = 0$$

$$x+3 = 0 \quad \text{or} \quad x-3 = 0$$

$$x = -3 \quad \quad \quad x = 3$$

The two numbers -3 and 3 separate the number line into three regions, A , B , and C .



Now we substitute the value of a test point from each region. If the test value satisfies the inequality, every value in the region containing the test value is a solution.

Region	Test Point Value	$(x+3)(x-3) > 0$	Result
A	-4	$(-1)(-7) > 0$	True
B	0	$(3)(-3) > 0$	False
C	4	$(7)(1) > 0$	True

The points in regions A and C satisfy the inequality. The numbers -3 and 3 are not included in the solution since the inequality symbol is $>$. The solution set is $(-\infty, -3) \cup (3, \infty)$, and its graph is shown.



PRACTICE

Solving a Polynomial Inequality

Think of it as a Quadratic equation ($<$ OR $>$ as a $=$ sign)

Solve equation

Plot answers

Pick numbers from each A,B,C

if equation is true part of solution

if false not part of solution

find and write out solution set

Quadratic Formula

Quadratic Formula

A quadratic equation written in the form $ax^2 + bx + c = 0$ has the solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the equation $b = c$

Get bases same by putting a power or square root or fraction

Cross out bases

Exponents become base

Solve the equation $b = c$ (cont)

Solve equation

Sometimes doesn't look like example, general it has 2 numbers raised to a power with an equal sign between

Graphing inverse

Look at $>$ or $<$ sign, determines what part of parabolas are the answer

Find the inverse

plot parabola from solved equation " $y = \dots$ "

Make a table of points

f and g functions

replace f and g

Horizontal line test

perform operation in

middle

$$(g \circ f)(x) = g(f(x))$$

intersects more than once, not a function

Didn't go over

9.3-9.6

Vertex Formula

First, isolate the x -variable terms by subtracting c from both sides.

$$y = ax^2 + bx + c$$

$$y - c = ax^2 + bx$$

Next, factor a from the terms $ax^2 + bx$.

$$y - c = a\left(x^2 + \frac{b}{a}x\right)$$

Next, add the square of half of $\frac{b}{a}$, or $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$, to the right side inside the parentheses. Because of the factor a , what we really added was $a\left(\frac{b^2}{4a^2}\right)$, and this must be added to the left side.

$$y - c + a\left(\frac{b^2}{4a^2}\right) = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right)$$

$$y - c + \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

Simplify the left side and factor the right side.

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

Add c to both sides and subtract $\frac{b^2}{4a}$ from both sides.

Compare this form with $f(x) = a(x-h)^2 + k$ and see that h is $-\frac{b}{2a}$, which means that the x -coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is $-\frac{b}{2a}$.

Vertex Formula

The graph of $f(x) = ax^2 + bx + c$, when $a \neq 0$, is a parabola with vertex

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

C

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